

Hierarchical Resource Allocation in Femtocell Networks using Graph Algorithms

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Abstract—This paper presents a hierarchical approach to resource allocation in open-access femtocell networks. The major challenge in femtocell networks is interference management which in our system, based on the Long Term Evolution (LTE) standard, translates to which user should be allocated which physical resource block (or fraction thereof) from which femtocell access point (FAP). The globally optimal solution requires integer programming and is mathematically intractable. We propose a hierarchical three-stage solution: first, the load of each FAP is estimated considering the number of users connected to the FAP, their average channel gain and required data rates. Second, based on each FAP's load, the physical resource blocks (PRBs) are allocated to FAPs in a manner that minimizes the interference by coloring the modified interference graph. Finally, the resource allocation is performed at each FAP considering users' instantaneous channel gain. The two major advantages of this suboptimal approach are the significantly reduced computation complexity and the fact that the proposed algorithm only uses information that is already likely to be available at the nodes executing the relevant optimization step. The performance of the proposed solution is evaluated in networks based on the LTE standard.

I. INTRODUCTION

Each new generation of wireless communication systems promises higher quality of service to a larger number of users. This requires new systems to find greater efficiencies in use of the scarce resources, especially, the radio spectrum. In the cellular context, an effective approach is to reduce the distance between the transmitter and the receiver, in turn reducing the transmit power and increasing the frequency reuse factor. Coupled with the observed increase in the indoor data activity [1], femtocells have been proposed as user-deployed small base stations to improve indoor coverage. The idea is that the user equipment (UE) would hand off from the cellular base station to the femtocell access point (FAP) installed at home or an office when operating indoors or when it is close enough to the FAP. The potential increase in the system capacity, however, encourages investigations into deploying femtocells to also service any set of opportunistic users which "see" the FAP (generally called open access) [2].

For FAPs to service outdoor users, they must transmit adequate power to provide coverage to a larger area. This, in turn, results in higher interference if neighbouring FAPs share a channel. In a multi-transmitter environment, such interference

would dominate performance. Hence, several methods have been proposed in literature to reduce interference. They mainly fall into two categories: power auto-configuration e.g., [3] and cognitive and dynamic spectrum allocation [4]–[6] (amongst many references). However, these works either require a lot of information to execute the relevant algorithm or require completely decentralized decision making that leads to interference or long convergence times to a solution.

In this paper, we focus on interference avoidance while allocating resources to meet users' minimum rate demands. Our analysis and design is in the context of femtocell-assisted networks based on the Long Term Evolution (LTE) standard. Here, resource allocation is defined in terms of connecting users to FAPs, the allocation of physical resource blocks (PRBs) to FAPs based on the users it supports and then to users. Clearly, solving this resource allocation problem to obtain the globally optimal solution requires integer programming and is, essentially, impossible in practice. In response, we present a partially decentralized and hierarchical resource allocation process. Each step in the process uses only local information, i.e., information is likely to be easily obtained at the node executing the step. Since LTE allows for reallocation of PRBs every millisecond [7], we allow for fractional allocation of PRBs to users; the fraction indicates the fraction of time the user is allocated the PRB.

The overall algorithm comprises three steps: (i) deciding the PRB requirements at FAPs based on users' data requirements and *average* channel gains; (ii) allocating specific PRBs to FAPs based on coloring an interference graph; (iii) FAPs determining max-min user allocations based on their required data rates. This work is presented in the downlink, however, similar work can be done in the uplink with different power constraints introduced to the problem. The key contribution of this paper is, therefore, a partially distributed algorithm that provides effective resource allocation and can be scaled to a large number of FAPs and users.

The rest of the paper is organized as follows: In Section II, we describe the system model and formulate the global optimization problem. The proposed sub-optimal solution is presented in Section III. The simulation results are performed in LTE standard presented in Section IV. The paper concludes with a discussion of the results in Section V.

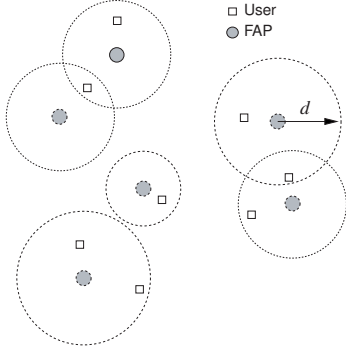


Fig. 1. Random distribution of FAPs in a femtocell Network.

II. SYSTEM MODEL AND THE OBJECTIVE

Figure 1 illustrates the femtocell network under consideration. We assume a single circular cell and independent uniform distribution for the location of all users and FAPs. A user connects to a single FAP only. We do not consider the basestation (BS) since it is generally provided an orthogonal frequency/PRB allocation; our model considers only users using FAPs¹. Each femtocell access point is at the centre of its coverage area. In the figure, d is the radius of the coverage area which depends on the environment and the FAP transmit power. Due to random geographic distribution of femtocells, some of them might interfere. In other words, some users might be in a location covered by more than one FAP.

The system comprises K users, and L FAPs are assumed according to a chosen user and FAP density. The physical channel between FAPs and users is modelled as frequency selective Rayleigh fading with average power determined by distance attenuation and large scale fading statistics. The total bandwidth is B subdivided into N sub-channels or PRBs. User k has a minimum data requirement of R_k bits per second (bps). To achieve fairness, we aim to maximize the minimum proportional rate, i.e., max-min of the achieved rate over all the users normalized by their required data rates. For convenience, the rate achieved on a specific channel is assumed to be given by the Shannon capacity, though more practical modulations can be easily accounted for using the corresponding gap to capacity [8].

Under this setting, the general form of the optimization problem is:

$$\begin{aligned} & \max_{p_{k,n}^{(l)}, S_l} \min_k \frac{1}{R_k} \left[\sum_{n=1}^N \frac{B}{N} \log_2 \left(1 + \frac{p_{k,n}^{(l)} h_{k,n}^{(l)}}{\sum_{i \neq l} p_{k,n}^{(i)} h_{k,n}^{(i)} + \sigma^2} \right) \right] \\ & \text{subject to} \quad \sum_{n=1}^N \sum_{k \in S_l} p_{k,n}^{(l)} \leq P_{max}, \quad l = 1, 2, \dots, L \\ & \quad p_{k,n}^{(l)} \geq 0, \quad \forall k, n \\ & \quad \bigcup_{l=1}^L S_l = K, \quad S_i \cap S_j = \emptyset \quad i \neq j \end{aligned} \quad (1)$$

¹If the BS allocation is not orthogonal, it can be easily incorporated as another transmitting node in the network with its own power budget.

where S_l is the set of users connected to and being serviced by FAP l . $p_{k,n}^{(l)}$ and $h_{k,n}^{(l)}$ are, respectively, the assigned power and channel power gain from FAP l to user k on subchannel n . B/N is the bandwidth and σ^2 the noise power on each subchannel². The first constraint is on the total transmit power of each FAP, while the second ensures non-negative powers. The third constraint ensures all users are connected to an FAP. Finally, the sets, $\{S_l\}_{l=1}^L$ are disjoint since each user is serviced by one and only one FAP at a time.

To solve this problem, we need to find the optimal S_l and $p_{k,n}^{(l)}$ leading to the max-min normalized rates. There are three issues that make finding the solution to this optimization problem impossible in practice: one, the assignment of users to FAPs makes this an intractable integer programming problem; two, the problem is non-convex and hence hard to solve; three, solving this optimization problem requires knowledge of all subchannels for all the users to all the FAPs. Getting this information to a central server would involve a huge overhead. Essentially, a resource allocation scheme based on perfect knowledge of channels is infeasible in a femtocell network.

III. PROPOSED HIERARCHICAL SOLUTION

This section presents the proposed algorithm that makes a series of reasonable, if sub-optimal, simplifications. First, we set the user-FAP connections by forcing each user to request service from the one FAP which offers the highest long term *average* received power (based, e.g., on a pilot and large-scale fading). This addresses the first issue raised above and is a significant step in making useful solutions feasible. We then decompose the problem of resource allocation into three parts: (i) load estimation, (ii) PRB allocation among FAPs and (iii) resource allocation among users at each FAP. Two of these phases are done locally at the FAP with small number of users and/or subchannels. This hierarchical decomposition enables us to take advantage of locally optimal algorithms.

A. Phase 1: Load Estimation

The FAP is assumed aware of the desired rate and *average channel power* for all users that it serves. Based on this information, the FAP *roughly estimates* its load in the form of total number of subchannels required by its users. At this phase, the objective at each FAP is to minimize the total number of required subchannels. This problem can be formulated as a convex optimization problem at each FAP as:

$$\begin{aligned} & \min_{w_k, P_k} \quad \sum_{k \in S_l} w_k \\ & \text{subject to} \quad w_k \frac{B}{N} \log_2 \left(1 + \frac{P_k H_k}{w_k \sigma^2} \right) \geq R_k, \quad \forall k \in S_l(2) \\ & \quad \sum_{k \in S_l} P_k \leq P_{max}, \\ & \quad P_k \geq 0, w_k \geq 0, \quad \forall k \in S_l \end{aligned}$$

where w_k is the number (can be a fraction) of subchannels FAP l assigns to user $k \in S_l$. P_k and H_k are the total power

²The terms ‘‘PRB’’ and ‘‘subchannels’’ are used interchangeably.

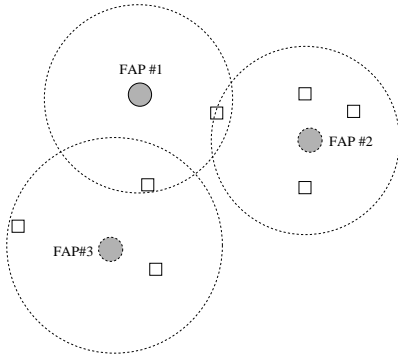


Fig. 2. FAP network with 3 FAPs. $N_1 = 1, N_2 = N_3 = 3$.

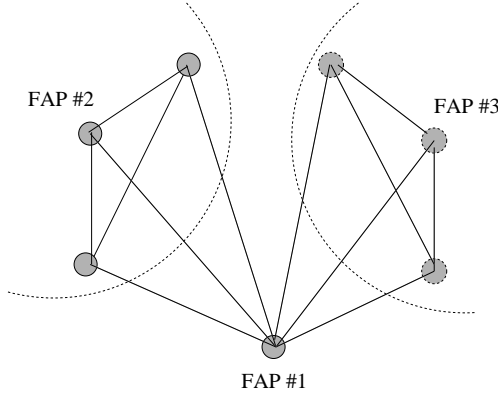


Fig. 3. Interference graph corresponding to Figure 2.

allocated to user k and the average channel gain seen by user k respectively. FAP l sets $N_l = \sum_{k \in S_l} w_k$ as the total number of subchannels it requires.

B. Phase 2: Channel Allocation Among FAPs Using Graphs

In this phase, the objective at the server is to assign subchannels to FAPs *proportional to their estimated load* while minimizing the interference among them. The FAPs report their requested loads, $\{N_l\}_{l=1}^L$ to a central server. Given N PRBs, if $N \geq \sum_{l=1}^L N_l$, each FAP is easily satisfied; however, this is unlikely. The server is assumed to know the interference graph based on *large-scale* statistics, i.e., it knows which FAPs potentially interfere with each other. The problem of channel assignment among FAPs can then be addressed by coloring the interference graph; here, each color corresponds to one subchannel. To take into account the load of FAPs, we modify the interference graph as follows: each FAP l is represented by N_l nodes (forming a complete subgraph). An edge connects two nodes if FAPs potentially interfere. The problem of channel assignment then becomes a graph coloring problem where two interfering nodes should not be assigned the same color. An example of a three FAP network is illustrated in Fig. 2. FAP #1 potentially interferes with FAP #2 and FAP #3. The corresponding interference graph and the coloring are illustrated in Figs. 3 and 4.

For arbitrary graphs, graph coloring is an NP hard problem. We adopt the heuristic algorithm proposed by Br  l  z [9]; at

every iteration, the vertex which is adjacent to the greatest number of differently-colored neighbours is colored with a new color, if necessary (until colors are exhausted). Note that by creating N_l nodes for FAP l , this process achieves an approximate proportional fairness in the case the server cannot meet the demand of $\sum_l N_l$ interference-free subchannels.

It is worth noting that optimal coloring of graphs is possible with low complexity if the interference graph is sparse such that each node is connected to at most N nodes where N is the total number of channels. Such graphs can be optimally colored with a modified Breadth First Search (BFS) algorithm with complexity of $\mathcal{O}(|V| + |E|)$ where $|E| = \alpha|V| = \mathcal{O}(|V|)$ where $|E|$ and $|V|$ are the cardinality of edges and vertices respectively and α is a scalar. Furthermore, due to the small coverage area of each femtocell, we assume FAPs are either interfering or not. However, by changing the interference graph into a weighted graph, the problem of interference reduction can also be formulated as max K -Cut problem where N is the number of available subchannels. The goal then is to find N groups of nodes where each group has the lowest edge weight among them and can be assigned the same color.

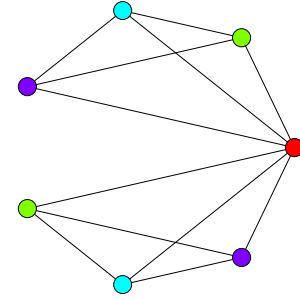


Fig. 4. Interference graph coloring corresponding to Figure 2. Minimum number of colors is 4, with both optimal and suboptimal coloring algorithms.

C. Phase 3: Resource Allocation Among Users

At the end of Phase 2, the FAPs have been assigned their subchannels. Each FAP can now allocate these resources amongst the users it serves. At each FAP, the problem is now reduced to maximizing the minimum rate for each user subject to their requested data rate. The FAP is assumed to know the channel state information for the users it serves. Let \bar{N}_l be the number of subchannels assigned to FAP l (not necessarily equal to N_l , the number of subchannels requested). At this FAP, the resource allocation problem is formulated as:

$$\begin{aligned}
 & \max_{p_{k,n}, c_{k,n}} \min_k \frac{1}{R_k} \frac{B}{N} \sum_{n=1}^{\bar{N}_l} c_{k,n} \log_2 \left(1 + \frac{p_{k,n} h_{k,n}}{\sigma^2} \right) \\
 & \text{subject to} \quad \sum_{n=1}^{\bar{N}_l} \sum_{k \in S_l} p_{k,n} \leq P_{max}, \quad p_{k,n} \geq 0, \quad \forall k, n \\
 & \quad \quad \quad \sum_{k \in S_l} c_{k,n} = 1, \quad c_{k,n} \geq 0 \quad \forall n
 \end{aligned} \tag{3}$$

where $c_{k,n}$ is the fraction of subchannel n (using time-division as allowed by LTE) allocated to user k .

This is a standard convex optimization problem easily solved for the time fraction and assigned powers. An even simpler alternative is to divide power equally amongst the \bar{N}_l subchannels, leading to a linear program.

To summarize the steps of the algorithm, the FAP reports its desired subchannel allocation to a central server based on long-term statistics. The server then uses the interference graph to assign subchannels in an interference free manner. Finally, the FAPs use their allocation optimally (locally) to provide resources to users. At each stage, the problems being solved use only local knowledge.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed scheme with simulations. The simulation setup is the LTE standard closely following [10]. The path loss between the FAP and the user accounts for indoor and outdoor propagation:

$$PL = 38.46 + 20 \log_{10}(d_{in}) + 37.6 \log_{10}(d) + L + L_s$$

where d_{in} is the distance between the FAP and the external wall or window and has a uniform distribution between 1m and 5m; L is the penetration loss and is set to 10dB and 3dB (with equal probability) for an external wall and windows respectively; L_s accounts for shadowing and is modeled as a log-normal random variable with standard deviation of 10dB. Finally, assuming Rayleigh fading, the instantaneous power of the received signal is modeled by an exponential random variable with the mean equal to the average receive power [11]. The receiver noise power spectral density is set to -174dBm/Hz with an additional noise figure at the receiver.

The downlink transmission scheme for an LTE system is based on OFDMA where the available spectrum is divided into multiple subcarriers each of bandwidth 15kHz. Resources are allocated to users in PRBs of 12 subcarriers; hence the bandwidth of each subchannel is 180kHz and is used as the signal bandwidth BW in calculating the noise power. Each PRB is allocated to a user at a time for a *subframe* duration of 1ms. Here we consider the maximum (20MHz) bandwidth corresponding to 100 PRBs of which N PRBs are allocated to the femtocells, i.e., $\sum_l \bar{N}_l \leq N$. A key assumption in the simulations is that since a PRB comprises 12 subcarriers, each PRB experiences an independent fade, i.e., we assume that the multipath environment is such that the fading is effectively flat for the 12 subcarriers in a PRB, but rich enough to yield an independent fade on each PRB.

Table I lists the parameters used in the simulations, unless otherwise specified.

The interference graph is generated based on the distance between each pair of FAPs. If the distance $D \leq 2d$, where d is the radius of the coverage circle of one FAP, the pair are connected in the graph and are assumed to interfere with each other. In practice, whether two FAPs interfere can be more accurately estimated by each FAP and reported to the central server.

TABLE I
SIMULATION PARAMETERS

Parameters	Value
Carrier frequency	2 GHz
Channel bandwidth	20 MHz
Carrier spacing	15 kHz
Resource Block	180 kHz
Total Number of PRBs	100
N	50
Transmit Power	20dBm
Antenna gain	0dB
Antenna Configuration	1 x 1
Noise Figure in UE	10dB
Minimum distance	1 meter from FAP
Penetration Loss (wall/window)	10dB/3dB
d	15m

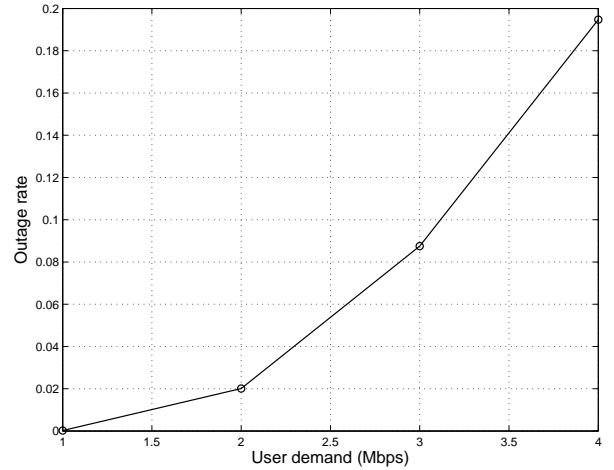


Fig. 5. Outage versus user rate demand in high FAP density network of 1 per circle of 5.5m radius. Cell radius = 100m.

Outage versus demand: Figure 5 plots the outage rate versus the user required data rate (demand). Here outage is defined as the fraction of users who do not receive 80% of their requested rate. The cell radius is set to 100m and the FAP density λ is one per 100 m^2 equivalent to 1 per radius of 5.5m. The same metric in a low FAP density network is shown in Figure 6 where λ is 1 per 1000 m^2 equivalent to 1 FAP per radius of 18m. In both cases, user density is chosen as 4λ equivalent to 4 users per FAP *on average*. All the users have equal demand. The results are averaged over 100 different user and FAP locations and 10 channel realizations per set of locations.

Comparing the two figures, the outage with high FAP density is higher for the same user demand. This is because, in both systems, the average number of users per FAP is constant. However, due to higher density of FAPs, they are more likely to interfere. Hence, the interference graph becomes dense in turn requiring more channels to color the graph. However, the total number of PRBs is limited ($N = 50$). As a result, each FAP gets smaller proportion of PRBs leading to higher outage.

It is worth mentioning that the key contribution here is the ability to wisely allocate resource allocation for a large number

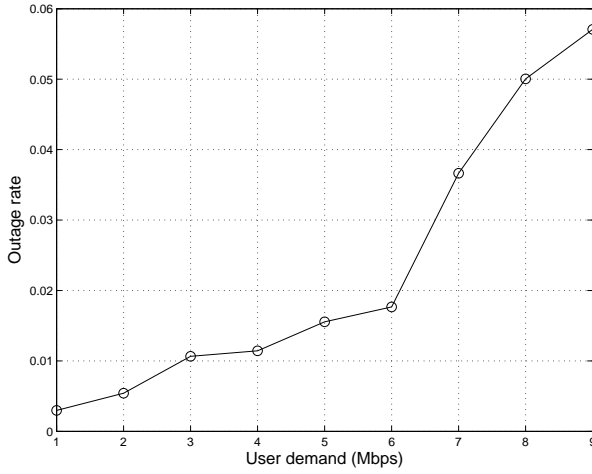


Fig. 6. Outage versus required data rate in low FAP density network of 1 per circle of 18m radius. Cell radius = 100m.

of users, FAPs and PRBs. In Fig. 5, $K = 1256$, $L = 314$ and $N = 50$. Given these large numbers, it is difficult to compare these results against any alternative scheme.

Achieved rate versus demand: Figure 7 plots the average minimum and maximum (over locations and channel realizations) user data rates in a low density network. Interestingly, the minimum achieved rate is approximately constant with increase in the demand whereas the maximum achieved rate decreases. This is because as the demand increases, so does the load of each FAP and hence N_l . Since each FAP is represented with N_l virtual nodes, the resulting graph gets dense. With the total number of PRBs fixed, each FAP receives a decreasing fraction of its requested load. Since the local resource allocation is in the form of max-min, the minimum data rate remains almost constant but the maximum achieved data rate decreases. Similar results were achieved for high density networks showing more fair distribution of resources with increase in the all users' demand.

V. CONCLUSION

In this paper, we proposed a hierarchical low complexity three-phase resource allocation scheme in open access femtocell networks. The main advantage of the proposed scheme is decomposing a complex non-convex optimization problem into several smaller convex problems with smaller sets of variables to optimize. The resulting hierarchical scheme is effective with a large problem size. The first phase of the proposed scheme is cell selection based on the long term averaged received power at each user and load estimation at each FAP based on its number of users and required service. The second phase attempts to reduce the interference among FAPs by coloring the modified interference graph considering FAPs load. Crucially, this step allocates resources to meet demands at individual FAPs. Finally, a convex problem in the form of maxmin is formulated at each FAP with small set of variables to maximize the minimum achieved data rate at each FAP.

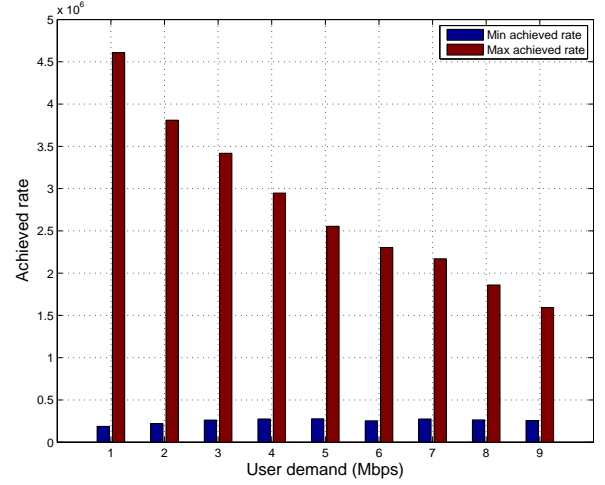


Fig. 7. Minimum and maximum user achieved data rate versus user demand. FAP density is 1 per circle of 18m. Cell radius = 80m.

At each step in the process, the node solving the related optimization problem requires information that it is likely to have. The central server only requires knowledge of the demands made by the FAP. Similarly, the FAPs require knowledge of local channels to users they serve; importantly, they do not require any global information. Note that the final step of resource allocation at the FAPs would, in practice, be solved in parallel.

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